

Probabilistic Graph Models (PGMs) for Feature Selection in Time Series Analysis and Forecasting

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Abstract—Time series or longitudinal analysis has a very important aspect in the field of research. Day by day new and better analyses are getting developed in this field. The main problem of the time series modeling is the presence of heteroskedasticity which was first identified as autoregressive conditional heteroskedastic (ARCH) effect by R. Engle (1969) [15] and then moderated by Bollerslev (1986) [7] in the more generalized form of generalized autoregressive conditional heteroskedastic (GARCH) models to explain the conditional dependence and can capture the systematizes evidence of the past variations of the time series variables. And on the other hand, J. Pearl (1985) [33] in the mid-eighties established the use of probabilistic graph models (PGMs), especially it's part where the causation cannot be reversed i.e. directed acyclic graphs (DAGs) as Bayesian Networks (BNs) to determine the conditional independence and is widely used in various fields of life with greater accuracy, precision, and fewer complexities. Fortunately, Bayesian Networks (BNs) are not used to to-date for the analysis of conditional dependencies or conditional independence analysis on the longitudinal data. This paper will review and summarize the uses of GARCH models and the uses of BNs in different fields and the responses of the researchers on the results accuracies and precision in comparison of the other available and applied analyses.

Keywords—Bayesian networks (BNs), directed acyclic graphs (DAGs), generalized autoregressive conditional heteroskedastic (GARCH) models, longitudinal analysis models, probabilistic graph models (PGMs), Time series models.

I. INTRODUCTION

Many time-series studies are made to the determination of the significant variables to be considered to get economies. The fundamental issue is to make a model closefisted for example with high exactness and least intricacy to go to the craving. (Beck, 1943) [6] There are many existing techniques for variable determination like hereditary calculations, jackknifing, step-wise relapse, Forward choice, and in reverse end strategies utilized in

relapse and time series examination (Andersen and Bro, 2010) [1] however these techniques are hard to apply and comprehend by the specialists with a feeble measurable foundation. Bayesian organizations (BNs) are the coordinated chart models (DAGs) built for the factors in a non-cyclic way and are created by Pearl (1985) [37] regularly utilized from the most recent thirty years for highlight choice in the field of hereditary qualities, virology, AI, and enormous information. This procedure isn't been utilized at this point in the time series examination.

II. GARCH MODELS(CONCEPTUAL CONTRIBUTION)

In an autoregressive moving average generalized autoregressive contingent heteroskedasticity (ARMA-GARCH) model, the condition for the severe stationarity, is the consistency of the greater request snapshots of the model. Typically, the consistency of the semi greatest probability assessor (QMLE) is demonstrated up to just the second request second condition which is called frail stationarity of a period series. This consistency result is new, in any event, for the univariate autoregressive contingent heteroskedasticity (ARCH) and Generalized autoregressive restrictive heteroskedasticity (GARCH) models. Additionally, the asymptotic ordinariness of the QMLE for the ARCH model is acquired under just the second-request snapshot of the genuine mistakes and the limited fourth-request snapshot of the contingent blunders. Under extra second conditions, the asymptotic ordinariness of the QMLE is additionally acquired for the ARCH and GARCH models and furthermore a predictable assessor of the asymptotic covariance. (Ling and McAleer, 2003) [33] Curve and GARCH models are the main devices in the investigation of time-series information, especially in monetary applications. These models are particularly helpful when the goal of the review is to break down and estimate unpredictability. This paper gives the inspiration driving the most straightforward GARCH model and delineates its helpfulness in looking at aggregate danger. (Engle R., 2001) [16] Curve models were presented by Engle (1982) [15], and Generalized ARCH or GARCH were proposed by Bollerslev (1986)

[7]. Curve process obsoleted on the grounds that GARCH supplanted ARCH with less boundaries and more precision. GARCH (1,1) would be adequate to deal with the greater part of the longitudinal investigations as it can catch the arranges proof of the past varieties. The Equation for GARCH (1,1) is as per the following:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

Where σ_t^2 is conditional volatility and ε_{t-1}^2 are squared error returns for the preceding period. α , β and ω would be always positive. ε_{t-1}^2 are obtained from the conditional mean equality that could be a random walk model $y_t = c + \varepsilon_t$ or AR (1) model $y_t = c + \gamma y_{t-1} + \varepsilon_t$, or any other ARMA or linear regression model $y_t = c + \gamma x_t + \varepsilon_t$. But generally, conditional mean formulas are kept simple as they can cause convergence problems in GARCH estimation [2]. Where y_t is the time series variable. Estimating the transient reliance in the second-request snapshots of information is significant for some issues in monetary econometrics. It is presently comprehensively recognized that monetary varieties move along with time across resources and markets. Perceiving this component through multivariate time series displaying prompts more reasonable models than isolated univariate models. According to a monetary perspective, it makes the way for better choice apparatuses in different regions, for example, resource valuing, portfolio determination, choice estimating, supporting, and hazard the board. Not at all like the beginning of the 1990s, few organizations, have now fostered the scientific capacities to utilize the econometric hypothesis according to a monetary point of view. Since the powerful paper of Engle [15], conventional time series instruments like autoregressive moving normal (ARMA) models [10] for the mean have been drawn out to generally resemble models for the difference. Summed up Autoregressive contingent Heteroskedasticity (GARCH) models are currently normally used to characterize and conjecture changes in the unpredictability of time series. [4]

III. GARCH MODELS(PRACTICAL APPLICATION)

In the new changed electric power industry, value expectation is exceptionally significant for the makers and buyers to figure and exploit their benefits. A summed up autoregressive restrictive heteroscedastic (GARCH) philosophy is utilized to gauge the upcoming power esteems. For the unprecedented variety in the power costs, the GARCH model is thought of, as proper for investigation of the time series information than other estimate models utilized regularly. The conjecture mistake is thought to be auto-corresponded autonomous from indicators with zero mean and steady fluctuation,

displayed by an Auto-Regressive (AR) process. In view of the beginning upsides of the boundaries of the model acquired by Eviews (programming), utilizing hereditary math to streamline the presentation. The outcomes show that the GARCH model is a superior foreseeing instrument with little root mean square blunders (RMS), Akaike Information Criteria (AIC) and a high R-square incentive at the future power cost in California liberated power market [26]. This research examined the volatility of exchange rates of Nigerian currency using simple GARCH (1,1), GJR-GARCH(1,1), EGARCH(1,1), APARCH(1,1), IGARCH(1,1), and TS-GARCH(1,1) models. Utilizing month to month data for the period of January 1970 to December 2007, Variability innovation and changeable characteristics are reviewed for the Naira/Dollar exchange rates. The effect of the versatility of the Foreign exchange market on assortment was broke down by showing results autonomously for the period before flexibility, the Fixed change standard period (January 1970-August 2006), and controlled float rule (September 2006 - December 2007). The outcomes from all of the models present that assortment is enduring. The outcome is something basically the same for the fair change scale period and controlled float rule. The revelations from all the anomaly models overruled the thought of the power sway. This is absence important to create by Nelson (1991). The APARCH model and GJR-GARCH model for the controlled drifting rate rule show the presence of a really basic peculiarity sway. The TSGARCH and APARCH models are considered to be more effective for such cases. [36]

IV. PROBABILISTIC GRAPH MODELS (PGMS) (CONCEPTUAL CONTRIBUTION)

A probabilistic graphical model (PGM) is a model for which a chart communicates the restrictive conditions among irregular factors. They are usually utilized in likelihood hypothesis, especially Bayesian insights and AI. By and large, probabilistic graphical models utilize a graphical portrayal regarding the base for fitting a dissemination over a multi-dimensional arrangement of factors and a chart that is a strong or incomplete portrayal of conditions that hold in the exact dispersion. Two kinds of graphical portrayals of dispersions that are regularly utilized are Bayesian organizations and Markov arbitrary fields. The two families cover the properties of factors and independencies, however they vary in independencies they can decipher and the factorization of the dispersion that they initiate as:

$$P([A, B, C, D]) = f_{(AB)}[A, B]f_{(AC)}[A, C]f_{(AD)}[A, D]$$

For some positive functions f_{AB}, f_{AC}, f_{AD} [32].

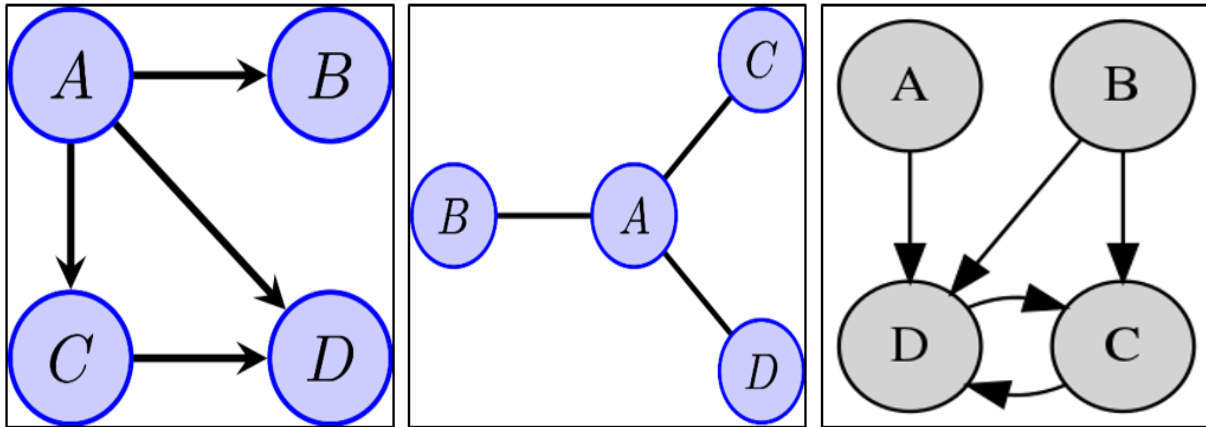


Fig.1 Probabilistic Graph Models (PGMs): a) Example of a directed acyclic graph on four vertices. b) An undirected graph with four vertices. c) An example of a directed, cyclic graphical model. [32]

V. PROBABILISTIC GRAPH MODELS (PGMs)
(PRACTICAL APPLICATION)

Because of the ascent of unlawful, public, and authoritative legal conditions including people lacking legitimate confirmation archives, age assessment of people groups has turned into a significant functional strategy for quite some time and criminological tests all around the world. The age of an individual is typically anticipated from the level of development of some chose body qualities utilizing measurable techniques. Be that as it may, their application might experience the ill effects of some hypothetical and genuine blunders, as of late guaranteed in the predefined writing. The target of this review is subsequently to give an elective approach to conquering these limitations, by rehashing the helpfulness of a probabilistic Bayesian methodology for age expectation. This methodology will permit to bargain in a completely clear technique with the frailty close the age gauging process and to deliver every one of the important information as back likelihood appropriation about the age of the individual under study. Besides, this likelihood dispersion can likewise be clearly utilized for registering. The likelihood that the examined individual is more youthful or more established than a given lawful age standard has specific law consideration. The primary advancement introduced by this exploration is the improvement of a probabilistic graphical model, or a Bayesian organization, for taking care of the central concern. The utilization of this sort of probabilistic model can work with fundamentally in the use of future systems [41]. The cell networks information according to alternate points of view is essential for atomic science. Probabilistic graphical models (PGMs) are helpful for getting significant natural dreams from the results. These models give an exact image of mind boggling cell networks by developing more straightforward models. Investigations dependent on notable standards for assessment of such models from information work with a model-based philosophy for assessment and finding. These practices

and their skills are demonstrated by a few existing applications to quality appearance information. (Friedman, 2004) [19]

VI. DIRECTED ACYCLIC GRAPHS (DAGS)
(CONCEPTUAL CONTRIBUTION)

In discrete math, especially diagram hypothesis, and software engineering, a coordinated non-cyclic chart (DAG) is a coordinated chart without any cycles. That is, it comprises of hubs and bolts (likewise called edges), with each edge guided starting with one hub then onto the next, to such an extent that those bearings won't ever frame a cycle. A coordinated diagram is a DAG if and provided that it can orchestrate the hubs as a straight association that is predictable with all edge bearings. DAGs have a few legitimate and computational uses in science (development, genealogies, and the study of disease transmission), humanism (reference organizations) to calculation (anticipating). [43]

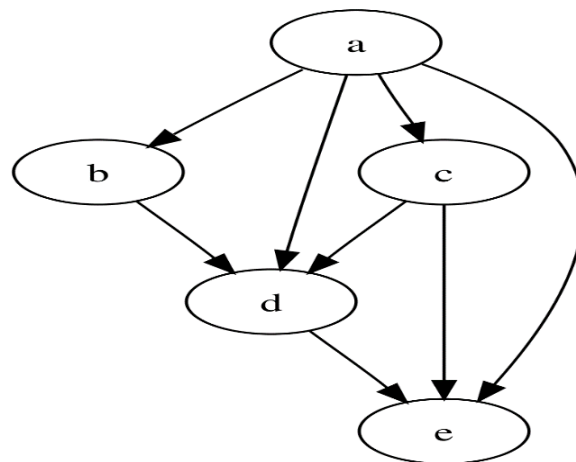


Fig. 2 Directed Acyclic Graph (DAG)

A diagram is shaped by hubs and edges interfacing hubs two by two, where the hubs can be any sort of item. On account of a coordinated diagram, each edge has a situating, starting with one hub then onto the next hub. A way in this diagram is an arrangement of edges having the property that the closure hub of each edge in the grouping is as old as beginning hub of the following edge in the succession; a cycle is produced assuming the beginning hub of its first edge rises to the completion hub of its last edge. A DAG is a coordinated diagram liberated from cycles [3].

VII. DIRECTED ACYCLIC GRAPHS (DAGs)
(PRACTICAL APPLICATIONS)

In this exploration, they utilized a data based philosophy to find the longitudinal and slacked relations among Bitcoin and other advantage modules. The picked strategy grants them to identify causal organizations dependent on the measures of identified basic and halfway connections, without trusting on deduced assumptions. Discoveries from the time series examination show that the Bitcoin market is very alone, and no particular advantages have a main influence in controlling the Bitcoin market. Nonetheless, they got proof of slacked relationship among Bitcoin and a few advantages, explicitly during the endure market territory of Bitcoin. This finding proposes that the blending among Bitcoin and other financial advantages is a nonstop practice that vacillates over the long run. They led an expectation of blunder fluctuation rot and observed that the effect of different advantages on Bitcoin north of a 20-day prospect doesn't clarify over 11% of all curiosities. (Ji et al., 2018) [28]

DAG investigation permits tending to the design of the information clarified association on time series information covariance, significant in giving minimized surmising in data taking care of and utilized here to inspect the lively relationship between these business sectors. From an applied position, this assurance of both the availability and course of impacts inside the advertising organization will be of exploratory use to actual sellers in the commercial center and be useful to talk about directors associated with looking at the conceivable part and accomplishment of another inferred arrangement. Subsequently the goal of this review was to underscore the meaning of the canal boat market and elaborate its part in the value investigation arrangement of the commodity promoting channel. The optional information is taken from United States Federal Grain Inspection Service over the period May sixth, 1999May third, 2001 set deduced, the overall significance of the canal boat market.

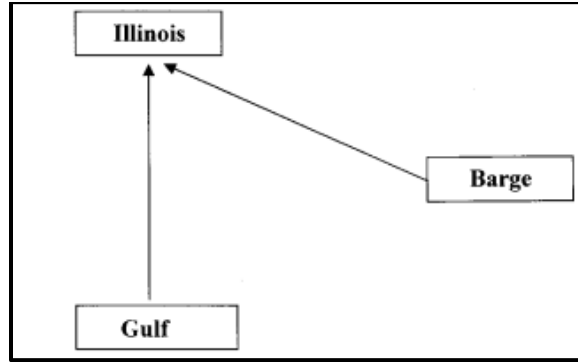


Fig. 3 A Directed Acyclic Graph for Causality and Price Discovery

The outcomes clarified that data from the Gulf market is critical in the value investigation methodology over all periods. While the universally impacted Gulf market doesn't genuinely influence the canal boat market that joins it to its neighborhood grain source at Illinois in said time, it is to some degree affected as the period changes. In any case, it is the Illinois market that is mediatory affected by the center east market. This impact hopes to wave by to the vehicle as stage changes, withdrawing the bearing of edge from the canal boat market influencing the Illinois market in a similar period, to the Illinois market profoundly influencing the flatboat market in the broad time frame. It shows up, accordingly, that for the more drawn out period both the neighborhood and worldwide business sectors altogether sway the flatboat market and jerks to these business sectors can fundamentally influence rates, adversely, or decidedly relying on where the jerks start. These jerks, regardless of whether they start from the Gulf or inland create additional changeability in the canal boat market, which could be destructive to revealing actual traders in this advertising channel. [24]

VIII. BAYESIAN NETWORKS (BNs) (CONCEPTUAL CONTRIBUTION)

The Bayesian organizations were developed by Judea Pearl (1985) to feature some exceptional DAGs properties in factual importance. It is once in a while emotional in nature of the info data. It covers the distinction among contributing and information methods of legitimizations. In the last part of the 1980s, Pearl's (1988) [37] Probabilistic Reasoning in Intelligent Systems and Neapolitan's (1989) [35] Probabilistic Reasoning in Expert Systems summed up their qualities and demonstrated them as a field of study. They depicted that a graphical model is an apparatus used to outwardly represent and work with the restrictive autonomy of the given factors. Two restrictively free factors straightforwardly affect one another. For instance for three factors displayed in the given graph (Fig.2), An is restrictively autonomous from C given B if $P(A/B,C) = P(A/B)$. Besides, the graphical model will show any intervening factors that different two contingent autonomous factors. These intervening factors

associate two restrictively free factors. A chart is made out of the hubs (factors) and the edges (impacts) which associate two hubs all at once.

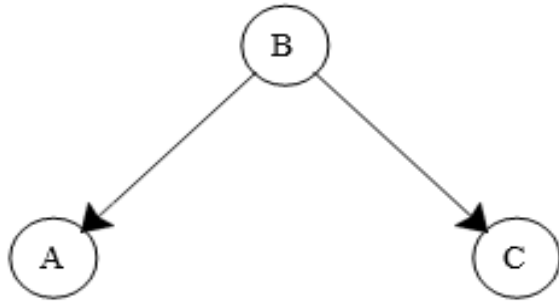


Fig. 4 Bayesian Network

In general, if we have X nodes named X_1, X_2, \dots, X_n , the joint probability function for the Bayesian network will be given as:

$$P(X) = \prod_{i=1}^n P(X_i / (Par(X_i)))$$

A Markov cover (MB) is minimal gathering of hubs which d-isolates hub v from any remaining hubs. A Markov cover (MB) of a hub is the gathering of hubs comprising of its folks, its youngsters, and different guardians of its kids. The Markov cover (MB) separates the centering hub autonomous from the remainder of the organization. A Markov cover for Node "A" can be found in the given outline covering its folks, its kids, and different guardians of its kids, and here, the gathering of guardians is a subgroup of the non-relatives components in light of the fact that the chart is non-cyclic.

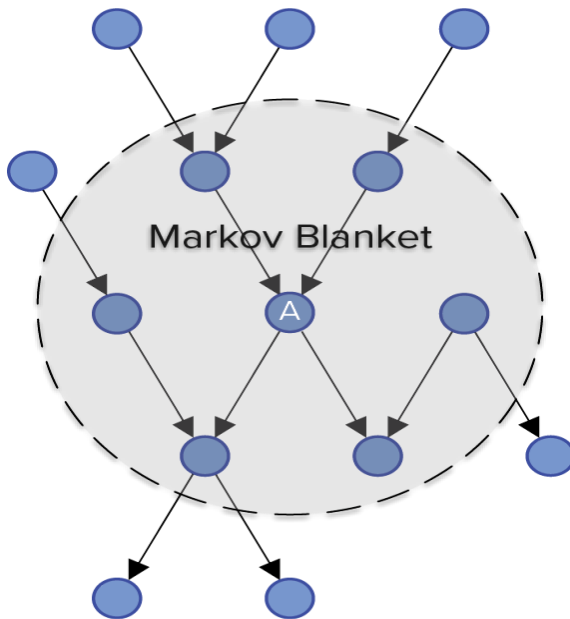


Fig. 5 Markov Blanket for node "A"

The Bayesian organization (BN) is a fortified model for probabilistic information show and deduction and is dynamically utilized in the field of reliance assessment. This paper introduced an audit of BNs that have been proposed for reliance assessment in the previous years. Writing is isolated from the perspective on the components of reliance assessment, i.e., equipment, information constructions, programming, and individuals. For each index, the arrangement and validation of a BN-based trustworthiness model are engaged. The expansive specialized strides for BN-based reliance assessment, including BN structure assessment, BN boundary assessment, BN extrapolation, and model approval, are analyzed. Existing openings and issues in reliance assessment with BNs are found, and a couple of future examination bearings for the scientists in this field are distinguished. [11]

IX. BAYESIAN NETWORKS (BNS) (PRACTICAL APPLICATIONS)

As indicated by the inadequacy of mistake tree examination in conditions, another strategy dependent on Bayesian organizations is proposed in this exploration. This paper was planned to change over blunder trees into Bayesian organizations, and clarify the likelihood of top-occasion, briefest ways, least cut sets, and shared data dependent on the Bayesian organizations. The assessment of the two strategies is completed for the "Daya Bay" thermal energy station. The discoveries show that the proposed method could improve and additional proof. [51]

This paper suggests the utilization of Bayesian Networks (BN) for application purposes in methodology demonstrating. Such a strategy can help in flying up demonstrating systems and making insignificant blunder models than displaying from start. The stream depends on the quantifiable technique models. The recommended component was tried on a model, coordinated from four certifiable techniques. The techniques were changed in accordance with save unrecognizability and high detect the likenesses. Truth be told, more extensive methods shift such a lot of that exemplary systems would yield practically irrelevant results as the majority of the positions would not be composed between methodologies. This issue can be settled by utilizing a characterized BP structure. For future work, it is proposed to underscore and demand suggestion strategy for foundation organization to further develop displaying methodology and assessment of the chose proposal draws near. They intended to play out a bunch of tests focused on at trying different things with proposal techniques on a few model sets. They additionally intended to extend the introduced strategies to utilize them in a joint displaying environment. (Bobek et al., 2013) [6]

The capacity of unpredictability causes judgment in the auto-body get together cycle has a significant impact in the accomplishment of the designing associations. However, it is more energizing to observe the methodology mistakes

associated with the agreeable sheet metal jobs dependent on little size informational indexes. Another Bayesian organizations (BN) demonstrating procedure under the circumstance of little informational collections is suggested. The really causative contacts are perceived dependent on the plotting of the instability affectability lattice. The coordinated effort impacts are seen by the restrictive joint data assessments. After the organization learning, the Bayesian method is utilized to get the restrictive probabilities by including earlier likelihood disseminations. The gauge of insightful execution in regards to sign number and log-chances clamor levels is additionally shown. A genuine section gathering case was used to clarify the entire cycle for fitting mistake examination. The assessed assessment showed the proposed BN method is applied and productive, in any event, when blemished ideas are distinguished and a medium-level commotion is available. (Liu and Jin, 2013) [34]

The plane designing business is attempting to apply imperative changes to its protection plot. This review introduced another design for building closing judgment on plane fixes' activities. It centers on the use of forecasts inside this overall setting to substitute healing and Precautionary Repairs drills for a scientific fix to lessen the charges of the maintenance subsidizing and to expand plane tasks. The vital point of this review is to communicate the Bayesian organization (BN) model as a valuable gauging approach. The exact use circumstance to figure brake clothing on the plane is built dependent on this strategy. The organization licenses assessed brake get from the plane working procedure. This model, alongside different models to frame figures for a long time of the plane gives a dynamic strategy for the situation of the plane, allowing the evaluation of different working procedures dependent on activity danger valuation and cost for every one of them contingent to the arranged checks. [17]

X. CONCLUSION

From the above discussion, we can conclude that many studies are made using Generalized Autoregressive Conditional Heteroscedastic (GARCH) models for time series data in many fields of life and getting better forecast efficiencies as compared to other longitudinal models, as well many studies used Bayesian networks (BNs), Directed Acyclic Graphs (DAGs) and Probabilistic Graph Models (PGMs) in many different situations including climate modeling, genes selection, and modeling, electricity transmission problems, machine learning problems, artificial intelligence application, flood forecasting, dam management, Floodwater storage management, Cellular modeling, etc. But no study has been found considered and evaluated by using Bayesian Networks (BNs) for longitudinal data. So there is a scope of research for using Bayesian Networks as a new recommendation for time series and longitudinal data modeling with more ease and precision.

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